Translation of the explanations of enclosure 2 of the search report of the European Patent Office ("Europäisches Patentamt") explaining the symbol letters stating the relevancy of the cited references:

Explanations

Column 1: Category of Named Documents

- X: Reference which by itself is considered to have special significance
- Y: Reference which in combination with another reference in the same category is considered to have special significance
- A: Technological background
- O: Disclosure not in writing
- P: In-between literature
- T: Theories or principles basic to the invention
- E: Older patent document published on or after filing date
- D: Reference cited in the patent application
- L: Reference cited for other reasons
- &: Member of same patent family, corresponding document

Column 2: Identification of document, specifying the relevant portions, if necessary

Column 3: Concerned claims
(In this column, the claims allocated to the relevant passages of column 2 are indicated.)

Column 4: Classification of Application (Int. Cl. 7)

Searched Fields (Int. Cl. 7)

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A NEW BIAS PARTITIONED SQUARE–ROOT INFORMATION FILTER AND SMOOTHER FOR AIRCRAFT FLIGHT STATE AND PARAMETER ESTIMATION

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Abstract

In this paper, a new bias partitioned square–root information filter (PSRIF) with an associated partitioned square–root information smoother (PSRIS) for aircraft flight state and parameter estimation is proposed. The new algorithm can not only improve the numerical robustness and precision of flight state estimation but also make the computation more efficient than augmented extended Kalman filter (EKF) or conventional square–root information filter and square–root information smoother (SRIF / SRIS). The results of simulated and actual flight test data computation of two types of Chinese aircraft show that the new method presented in this paper can give accurate estimates of flight state and parameter for high and low sampled rate and is much more numerical stable and efficient.

Introduction

Accurate estimation of aircraft motions from noisy or incomplete flight test measurements by optimal estimation theory is an important problem in the analysis of flight test experiments. The measurements may often contain significant errors which must be estimated before the flight data are used in any performance calculations. These problems can be implemented as state and parameter estimation problem of nonlinear system. Solution to the problem can be obtained by Extended Kalman Filter (EKF)[1, 2] and Maximum Likelihood (ML)[2] method. However, the ML method is not suitable to on–line processing of flight data and can consider the case with observation noise only. EKF may suffer from numerical ill–conditioning. In order to improve the numerical stability and computation efficiency, A new PSRIF / PSRIS for flight state and parameter estimation is proposed. The modified algorithm is based on a new aircraft flight state and parameter estimation model of separated bias from state vector other than conventional augmented state vector one. The new algorithm can not only improve the numerical robustness and precision of flight state estimation but also make the computation more efficient than augmented EKF or conventional SRIF / SRIS.

Nonlinear Model for Flight State Estimation

The six degrees of freedom nonlinear model for flight state estimation is presented as follows:

$$
\left.
\begin{aligned}
\dot{u} &= -qw + rv - g\sin\theta + a_x \\
\dot{v} &= -ru + pw - g\cos\theta\sin\varphi + a_y \\
\dot{w} &= -pv + qu - g\cos\theta\cos\varphi + a_z \\
\dot{\psi} &= (q\sin\varphi + r\cos\varphi)/\cos\theta \\
\dot{\theta} &= q\cos\varphi - r\sin\varphi \\
\dot{\varphi} &= p + q\sin\varphi\,tg\theta + r\cos\varphi\,tg\theta \\
\dot{h} &= u\sin\theta - v\cos\theta\sin\varphi - w\cos\theta\cos\varphi
\end{aligned}
\right. \qquad (1)
$$

In general, the measurements of input variables, $\mathbf{U} = [a_x, a_y, a_z, p, q, r]^T$ and observation variables $\mathbf{Z} = [V, \beta, \alpha, \theta, \varphi, h]^T$ are corrupted by scale factor errors, biases and random noises, where

$$
\left.
\begin{aligned}
a_x &= a_{x_m} + b_x + \eta_x \\
a_y &= a_{y_m} + b_y + \eta_y \\
a_z &= a_{z_m} + b_z + \eta_z \\
p &= p_m + b_p + \eta_p \\
q &= q_m + b_q + \eta_q \\
r &= r_m + b_r + \eta_r
\end{aligned}
\right\} \qquad (2)
$$

The observation equation of aircraft can be described as follows:

$$
\left.
\begin{aligned}
V_m &= (1+\lambda_V)\sqrt{u^2+v^2+w^2} + b_V + \xi_V \\
\beta_m &= (1+\lambda_\beta)\tan^{-1}[(v + rx_\beta - pz_\beta)/u] + b_\beta + \xi_\beta \\
\alpha_m &= (1+\lambda_\alpha)\tan^{-1}[(w - qx_\alpha + py_\alpha)/u] + b_\alpha + \xi_\alpha \\
\theta_m &= \theta + b_\theta + \xi_\theta \\
\varphi_m &= \varphi + b_\varphi + \xi_\varphi \\
h_m &= (1+\lambda_h)h + b_h + \xi_h
\end{aligned}
\right\} \qquad (3)
$$

The system described by equations (1)~(3) forms a set of nonlinear dynamic equations of the form:

$$\dot{\mathbf{X}}(t) = \mathbf{f}\ (\mathbf{X}(t),\ \mathbf{U}_m(t),\ \mathbf{b},\ \eta(t)) \qquad (4)$$

$$\mathbf{Z}_m(t) = \mathbf{g}\ (\mathbf{X}(t),\ \mathbf{U}_m(t),\ \mathbf{b}) + \xi(t) \qquad (5)$$

where: $\mathbf{X} = [u,\ v,\ w,\ \psi,\ \varphi,\ \theta,\ h]^T$

$$\mathbf{b} = [b_x, b_y, b_z, b_p, b_q, b_r, b_V, b_\beta, b_\alpha, b_\theta, b_\varphi, b_h, \lambda_V, \lambda_\beta, \lambda_\alpha, \lambda_h]^T$$

Thus, our problem can be stated as follows: Given the nonlinear model (4), (5) and a set of noisy input and output measurements, estimate the system state $\mathbf{X}$ and parameter $\mathbf{b}$.

New Bias Partitioned Square–Root Information Filter / Smoother

In order to estimate the state and unknown parameters, in general, one can form an augmented state model, that is, set

$$Y = \begin{bmatrix} X \\ b \end{bmatrix} \tag{6}$$

the equations (4) and (5) can be rewritten as an augmented state formulation:

$$\dot{Y}(t) = f_A\ (Y(t),\ U_m(t),\ \eta(t)) \tag{7}$$

$$Z_m(t) = g\ (Y(t),\ U_m(t)) + \xi(t) \tag{8}$$

Taking account for the unknown bias error vector **b** be supposed to be constant, linearization and discretization of equations (7) and (8) around a 'nominal' flight condition yields the following set of linear discrete–time equations of the separated state and bias form:

$$\left.\begin{aligned} x(k+1) &= \Phi_x(k+1,k)x(k) + \Phi_{xb}(k+1,k)b + \Gamma_x(k+1,k)\eta(k) \\ b_{k+1} &= b_k \end{aligned}\right\} \tag{9}$$

$$z(k+1) = G_x(k+1)x(k+1) + G_b(k+1)b + \xi(k+1) \tag{10}$$

where **x**, **z** represent the deviations from their reference values. Thus, the dimension of system will be reduced to the larger one of n_x or n_b other than n_x+n_b. This would require fewer numerical operations than the augmented state one and may avoid the numerical ill–conditioning caused by consideration of biases as the augmented state. The process noise η and measurement noise ξ sequences are assumed to be independent, zero mean and Gaussian with covariance Q and R, respectively.

The PSRIF / PSRIS based on the Eqs.(9) and (10) can be derived and summarized as follows:

Time update: Choose an orthogonal transformation $\bar{T}_k$, such that:

$$\bar{T}_k \begin{bmatrix} -\hat{S}_x\Phi_x^{-1}\Gamma_x & \hat{S}_x\Phi_x^{-1} & \hat{S}_{xb} - \hat{S}_x\Phi_x^{-1}\Phi_{xb} \\ S_w & 0 & 0 \end{bmatrix} = \begin{bmatrix} \bar{S}_w & \bar{S}_{wx} & \bar{S}_{wb} \\ 0 & \bar{S}_x & \bar{S}_{xb} \end{bmatrix}$$

where $\bar{T}_k$ denotes a Householder transformation or modified weighted Gram – Schmidt transformation such that $\bar{S}_w$ is an upper triangular matrix.

Measurement update:

$$\hat{T}_x \begin{bmatrix} \bar{S}_x & \bar{S}_{xb} & 0 \\ S_v G_x & S_v G_b & S_v z \end{bmatrix}_{k+1} = \begin{bmatrix} \hat{S}_x & \hat{S}_{xb} & \hat{z}_x \\ 0 & \hat{G}_b & \hat{z} \end{bmatrix}_{k+1} \tag{12}$$

$$\hat{T}_b \begin{bmatrix} \hat{G}_b(k+1) & \hat{z}(k+1) \\ \hat{S}_b(k) & \hat{z}_b(k) \end{bmatrix} = \begin{bmatrix} \hat{S}_b(K+1) & \hat{z}_b(k+1) \\ 0 & e(k+1) \end{bmatrix} \tag{13}$$

The optimal correction $x(k+1)$ is calculated as follows:

$$\hat{x}(k+1) = \hat{S}_x^{-1}(k+1)[\hat{z}_x(k+1) - \hat{S}_{xb}(k+1)\hat{b}(k)] \tag{14}$$

$$\hat{b}_{k+1} = \hat{S}_b^{-1}(k+1)\hat{z}_b(k+1) \tag{15}$$

The estimate of $X(k+1)$ and / or the estimated covariance matrix are

$$\hat{X}(k+1) = \bar{X}_{nom} + \hat{x}(k+1) \tag{16}$$

$$\hat{P}_x = [\hat{S}_x^{-1}(k+1)L(k+1)][\hat{S}_x^{-1}(k+1)L(k+1)]^T \tag{17}$$

where: $L(k+1) = [I \ \vdots\ -\hat{S}_{xb}(k+1)\hat{S}_b^{-1}(k+1)]$

Noting that the filtered estimation of **b** is also the smoothed one, then the estimation of **b** can be eliminated from the smoothing process, the dimension of the smoother can be reduced. The partitioned square–root information smoother (PSRIS) is, for k = N, N−1, ···, 0, recursively select orthogonal transformation T_x^* such that:

$$T_x^* \begin{bmatrix} \bar{S}_w + \bar{S}_{wx}\Gamma_x(k) & \bar{S}_{wx}\Phi_x(k) & \bar{S}_{wx}\Phi_{xb}(k) + \bar{S}_{wb} & 0 \\ S_x^*\Gamma_x(k) & S_x^*\Phi_x(k) & S_x^*\Phi_{xb}(k) + S_{xb}^* & z_x^*(k+1) \end{bmatrix}$$
$$= \begin{bmatrix} S_w^*(k+1) & S_{wx}^*(k+1) & S_{wb}^*(k+1) & z_w^*(k+1) \\ 0 & S_x^*(k) & S_{xb}^*(k) & z_x^*(k) \end{bmatrix} \tag{18}$$

the smoothed state estimate and the smoothed state covariance are obtained from:

$$X_k^* = S_x^{*-1}(k)[z_x^*(k) - S_{xb}^*(k)b^*] \tag{19}$$

$$P_x^* = [S_x^{*-1}(k)L(k)][S_x^{*-1}(k)L(k)]^T \tag{20}$$

where: $L(k) = [I \ \vdots\ -S_{xb}^*(k)S_b^{*-1}(k)]$

Simulation and Application

The new algorithm presented in this paper has been applied to the flight state and parameter estimation with simulated and actual flight test data. The results of simulation and application to two types of Chinese aircraft show that the new algorithm can give accurate estimate and is much more stable and efficient than EKF or conventional SRIF / SRIS. The results of estimation are given in Fig.1 and Fig.2 with the sample period of 1 / 32 and 1 / 10 second, respectively. It is obvious that the results are satisfactory both for higher and lower sampling rates.

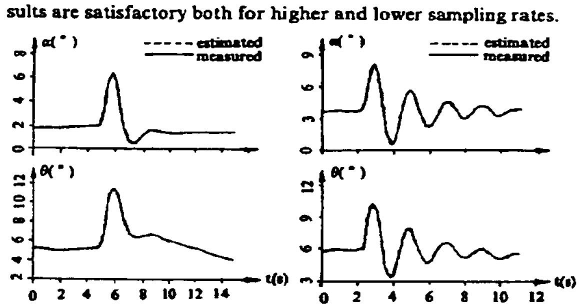

Fig.1 Results of state estimate(1 / 32s) Fig.2 Results of state estimate(1 / 10s)

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